A TREATISE ON PLANE CO-ORDINATE GEOMETRY: OR, THE APPLICATION OF THE METHOD OF CO-ORDINATES TO THE SOLUTION OF PROBLEMS IN PLANE GEOMETRY. PART I

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A Treatise on Plane Co-Ordinate Geometry: Or, the Application of the Method of Co-Ordinates to the Solution of Problems in Plane Geometry. Part I by M. O'Brien

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A TREATISE

ON

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APPLICATION OF THE METHOD OF CO-ORDINATES

TO

THE SOLUTION

OF

PROBLEMS IN PLANE GEOMETRY.

PART I.

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PREFACE.

THE subject treated of in the following pages is usually styled *Analytical Geometry*, but its real nature seems to be better expressed by the title *Co-ordinate Geometry*, since it consists entirely in the application of the Method of Co-ordinates to the Solution of Geometrical Problems.

Recd. 01-16-41. HBJ.

The present Treatise, in which we shall confine our attention to figures and curves in one plane, will consist of two parts: the first part is all that is at present published; it contains the application of the method of Co-ordinates to Right Lines, Circles, and Conic Sections. In the second part, the properties of Curves in general, with reference to Tangents, Asymptotes, Singular Points, Curvature, &c., will be investigated, without assuming a knowledge of the Differential Calculus on the part of the Student. A Historical Account of the subject, and a large collection of Problems will be added.

The complete analysis given in the Table of Contents renders it unnecessary to say much here respecting the plan pursued in the Treatise. In the first chapter the meaning of the signs = + and −, and the nature of negative and imaginary quantities, are fully explained, on principles which seem to combine generality and simplicity. The difficulties

which are supposed to beset the foundations of Algebra, are partly due to the indistinctness of the definitions usually given of algebraical symbols. In the subsequent chapters, the Author has endeavoured to adhere to a uniformity of method which, he hopes, will render it easy to acquire and retain a knowledge of the subject. He has also made use of symmetrical equations as much as possible, as there can be no doubt that many advantages are lost, and none gained, by want of attention to symmetry in analytical processes. The properties of Conjugate diameters are investigated by means of the angle called the Eccentric Anomaly in Astronomy, but the same properties are deduced without making use of this angle in Chapter XI. Several geometrical illustrations are given in notes, and various Problems and Examples, many of which are taken from the Senate-House Problems and other sources are added at the end of each chapter.

CAMBRIDGE,
March, 1844.

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