THE ANALYSIS AND SOLUTION OF CUBIC AND BIQUADRATIC EQUATIONS: FORMING A SEQUEL TO THE ELEMENTS OF ALGEBRA

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THE

ANALYSIS AND SOLUTION

OF

CUBIC AND BIQUADRATIC EQUATIONS;

FORMING A SEQUEL TO

THE ELEMENTS OF ALGEBRA,

AND AN INTRODUCTION TO THE

THEORY AND SOLUTION OF EQUATIONS OF THE HIGHER ORDERS.

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LONDON:
SOUTER AND LAW, 131, FLEET STREET.
1842.

PREFACE.

Considerable advances have of late been made in that department of analysis to which the theory and solution of Numerical Equations belongs; although, as is often the case with extensive innovations upon long-established practices, these improvements have met with general acceptance by but very slow degrees.

In a work on Equations, published in 1835, I endeavoured to embody the principal of the modern discoveries in this department of enquiry, in a form accessible to the algebraic student; and from the various public testimonies which have appeared in commendation of that endeavour, I am encouraged to hope that the book has in some degree contributed to awaken that spirit of examination into the pretensions and capabilities of the modern methods, which is now in pretty active exercise among British mathematicians; and from which it may be reasonably expected that these methods will be eventually perfected into the forms best suited to the actual demands of practical science.

On account of the variety of topics discussed in the publication just adverted to, all of them closely connected with the general doctrine of equations, it was found that brevity and compression were essential to the design of comprehending the enquiry within the compass of a single volume of moderate size and price.

The important theorems of STURM and HORNER, more especially the former, were thus perhaps too hastily disposed of; and many interesting details and developments omitted which might have placed the practical value of these theorems more conspicuously and more favorably before the reader.

But at that time the theorem of STURM was new: and I must acknowledge that I had but too imperfectly examined into its capabilities of practical improvement to justify my venturing upon any addition to what the author himself had delivered.* I have since more deli-

* Sturm's discovery was not known to me till my book on Equations had been some time at press, and was in great part printed. I believe the fame of it had not reached England at that time, as the publication of my own book seems to have furnished the first announcement of it in the English language. I may further mention, in order to escape the charge of hasty negligence, to which the admission in the text may expose me, that the first intimation I received of the theorem was in May, 1835, through the last sheet of an edition of the *Complément d'Algèbre*, then at press, and given by LACROIX to a friend of mine (Mr. SPILLER) at that time at Paris. LACROIX spoke somewhat disparagingly of the discovery; but I thought it of sufficient importance to justify the destruction of many pages of my MS. and the suspension of the work till the volume containing Sturm's paper should be published. This volume —a ponderous quarto of upwards of 1000 pages—was forwarded to me from Paris, by post, in July; and my own treatise was published in the following August. I trust it will not be inferred, from this explanatory note, that I consider any merit to be due to the mere importing of a discovery from one country to another: yet it may be well to have shown that the importation is not always unattended with inconvenience, anxiety, and expense.

berately considered the subject, and have offered some suggestions for facilitating the numerical process which the practice of the theorem requires. These suggestions will be found in the *Mathematical Dissertations*, recently published; and others of a different kind, but aiming at the same object, are given in the volume now submitted to the public.

In this volume I have endeavoured to unfold, with all the amplitude that can be desired, as much of the theory and solution of Numerical Equations as can reasonably be expected to enter into an ordinary course of mathematical education. I wish it to be regarded as a sequel to elementary Algebra; and as a comprehensive introduction to the more advanced volume on the *Theory of Equations of the higher orders*, a new and enlarged edition of which will be shortly published.

As the discoveries of STURM and HORNER are destined to become permanently incorporated among the doctrines of analysis, I have spared no pains to present them to the young student with all due copiousness of illustration and practical detail. The theoretical exposition of them is different from that delivered by the authors themselves, being more simple and elementary; whilst a larger number of practical applications of them have been given than any other work affords.

These numerical illustrations have involved me in considerable labour, as a glance at the closing sheets of the work will render sufficiently apparent.

This labour might however have been greatly reduced by the exercise of a little management, in framing the examples in favorable adaptation to the methods employed. I have not adopted any manœuvres of this kind; but have, on the contrary, been anxious to submit the new methods to the severest tests, by applying them to the most unfavorable and difficult examples I could find.

The plan that I have recommended at page 111 for conducting the operation for analysing an equation by the method of STURM, is different from, and somewhat longer than that proposed in the *Dissertations*. It is not to be preferred to this latter process on account of superior brevity—which it has not; but merely because its more elementary simplicity gives it a better claim to admission into an introductory work like the present.

The method of analysing an equation proposed at page 127, and so fully illustrated in the closing chapter, and by which the most laborious part of STURM's theorem is dispensed with, or rather exchanged for another operation, is I think deserving of examination, if not of general adoption. In equations of the more advanced degrees considerable advantage will often be derived from the examination of the leading *vertical* rows of signs in STURM's table, as well as of the *horizontal* rows; agreeably to the suggestions of the method adverted to. I have applied this method to the analysis of a very unpromising example at page 221; and have, for the