

**THE ELEMENTS OF
PLANE
TRIGONOMETRY**

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The Elements of Plane Trigonometry by H. N. Wheeler

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H. N. WHEELER


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BY
H. N. WHEELER, A.M.

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PREFACE.

THIS book originated in a desire to give to my private pupils a better idea of the trigonometric functions of obtuse angles than they could obtain from any book with which I was acquainted.

I believe that a student can get a comprehensive and thorough knowledge of trigonometry most quickly and easily, if, at the outset, such definitions are given to the trigonometric functions as will apply to all angles: with this idea for a basis, I have endeavored to prepare an elementary text-book for general use. By beginning with an explanation of the use of the negative sign as applied to lines and angles, and then giving general definitions to the trigonometric functions, I have been able to demonstrate all the fundamental formulas in a perfectly general yet simple manner.

While I have tried to present the subject from an elementary point of view, I have not lost sight of the fact, that, to most students, trigonometry is merely a stepping-stone to something higher; and for this

reason I have also tried to present the results in such a light as will make them effective tools for the student in his future work.

During the preparation and printing of the book, Prof. J. M. Peirce has kindly given me the benefit of numerous suggestions and criticisms, which have added greatly to its value. Appendix I. is entirely due to him.

Most of the examples contained in the book have been selected from other trigonometries, especially those of Todhunter, Snowball, and Hamblin Smith.

TABLE OF CONTENTS.

CHAPTER I.

DEFINITIONS AND FIRST PRINCIPLES.

	Page.
§ 1. Trigonometry defined	1
§ 2. Line regarded as traced by the movement of a point. Beginning and end of a line. Addition of lines defined,	2
§ 3. Direction of a line. Opposite directions indicated by signs + and -. Line completely represented by an expression involving the points at its extremities. Formulas for the sum of lines	2
§ 4. Degree of angle. Degree of arc. Quadrant defined	4
§ 5. Complement and supplement of an angle or arc defined	4
§ 6. Circular measure of an angle the ratio of intercepted arc to radius	5
§ 7. Circular measure equal to the length of the intercepted arc in the unit circle	5
§ 8. Angular unit of circular measure that angle whose intercepted arc is equal to the radius	6
§ 9. To change from circular measure to degree measure, and the converse. Angular unit of circular measure equal to $57^{\circ}.2957795$. Circular measure of 1° , $1'$, and $1''$	6
§ 10. Circular measure of 360° , 180° , 90° , and 45° , equal to 2π , π , $\frac{\pi}{2}$, and $\frac{\pi}{4}$ respectively	7
§ 11. Angle regarded as the measure of the rotation of a line,	7

§ 12. No limit to angular magnitude. Initial line. Terminal line	8
§ 13. Angle represented by three letters; the order of the letters. Angle represented by two letters. (<i>Peirce, Notation.</i>)	8
§ 14. Positive and negative rotation defined	9
§ 15. 1st, 2d, 3d, and 4th quadrants defined. Figure for an angle of each quadrant	9
§ 16. Addition of angles	10
§ 17. Remarks about positive and negative angles	10
§ 18. Angles φ and $\varphi \pm k 360^\circ$ bounded by the same lines	11
§ 19. Equations obtained by a general application of § 16	11
§ 20. Rules concerning the directions of certain lines	12
§ 21. Statement of a few principles of plane geometry to which it will be necessary to refer hereafter.	14
Examples	15, 16

CHAPTER II.

THE TRIGONOMETRIC FUNCTIONS.

§ 22. Triangle of reference formed by letting fall a perpendicular from any point of the terminal line upon the initial line. Figures. Signs of the sides of a triangle of reference	17
§ 23. The <i>Trigonometric Functions</i> defined to be the six ratios between the three sides of a triangle of reference. These ratios shown to be always the same for the same angle, and different for different angles. Reason for the name "Function"	18
§ 24. A name given to each of the trigonometric functions. Versed sine	20
§ 25. Table of signs of the different functions for an angle of each quadrant. Sin and cos vary only from -1 to $+1$; sec and csc, from ± 1 to $\pm \infty$; tan and ctn, from $-\infty$ to $+\infty$	20
§ 26. Sin, cos, and tan, reciprocals of csc, sec, and ctn respectively	21
§ 27. $\tan \varphi = \frac{\sin \varphi}{\cos \varphi}$, $\text{ctn } \varphi = \frac{\cos \varphi}{\sin \varphi}$	21
§ 28. Relations between sin and cos, tan and sec, ctn and csc,	22

TABLE OF CONTENTS.

vii

§ 29. Trigonometric functions represented by straight lines	23
§ 30. $\sin \varphi = \frac{1}{2}$ chord 2φ	26
§ 31. Relations between the functions of φ , $\varphi \pm k 360^\circ$, $180^\circ - \varphi$, $180^\circ + \varphi$, $360^\circ - \varphi$, and $-\varphi$	27
§ 32. Relations between the functions of φ , $90^\circ + \varphi$, $90^\circ - \varphi$, $270^\circ + \varphi$, and $270^\circ - \varphi$	28
§ 33. Complementary functions	29
Rule for remembering the relations between the functions of angles in the different quadrants	
§ 34. Functions of 0° and 360°	30
§ 35. Functions of 90°	31
§ 36. Functions of 180°	31
§ 37. Functions of 270°	32
§ 38. Functions of 30° and 60°	32
§ 39. Functions of 45°	33
§ 40. Functions of 120°	33
§ 41. Functions of 150°	34
§ 42. Functions of 135°	34
§ 43. Table containing the results of the last nine sections	35
§ 44. Increase and decrease of functions	36
§ 45. Inverse or anti functions	37
Examples	39-41

CHAPTER III.

THE FUNCTIONS OF THE SUM AND DIFFERENCE OF TWO ANGLES, AND FORMULAS DEDUCED FROM THEM.

§ 46. Formulas for \sin and \cos of $(\alpha \pm \beta)$ in terms of \sin and \cos of α and β ; general solution; figure for the case where α and β are acute and positive	42
§ 47. Figures to assist in showing the generality of the above solution	45
§ 48. Formulas for \tan and \cotn of $(\alpha \pm \beta)$ in terms of \tan and \cotn of α and β	46
§ 49. Four formulas deduced from \sin and \cos of $(\alpha \pm \beta)$	47
§ 50. Formulas for the functions of 2α	47
§ 51. Formulas for the functions of $\frac{1}{2}\alpha$	48
§ 52. Other forms for $\cotn 2\alpha$, $\csc 2\alpha$, $\tan \frac{1}{2}\alpha$, $\cotn \frac{1}{2}\alpha$, and from these, formulas for \tan of $(45^\circ + \frac{1}{2}\alpha)$ and $(45^\circ - \frac{1}{2}\alpha)$	48
§ 53. Twelve useful formulas deduced from the formulas obtained in §§ 46-51	49
Examples	51-53