THE ELEMENTS OF THE DIFFERENTIAL CALCULUS: FOUNDED ON THE METHOD OF RATES OR FLUXIONS, PART SECOND

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PART SECOND

BY

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PREFACE.

This small volume constitutes the *second part* of an elementary text-book on the Differential and Integral Calculus. The *first part* was printed by John Wiley & Son, New York, 1874.

The *third part* will contain chapters on the Evaluation of Indeterminate Forms; on Functions of two or more Independent Variables; and on the Application of the Differential Calculus to the Theory of Curves.

After the completion of the third part, it is our intention to incorporate the three parts in a single volume on the Differential Calculus; this will be published, and also a volume on the Integral Calculus, as soon as practicable.

J. M. R.
W. W. J.

U. S. Naval Academy, *October*, 1875.

CHAPTER IV.

SUCCESSIVE DIFFERENTIALS AND DERIVATIVES.

XII.

THE DIFFERENTIATION OF FUNCTIONS OF THE TIME.

65. When a variable quantity is expressed as a function of t (the time elapsed from a given instant or *origin* of time), its *rate* has a definite value for each instant (Art. 11), and is simply a function of the time. The processes already established for the differentiation of functions of x apply likewise to those of t; thus,—

$$d\left(\frac{a}{x}\right) = -\frac{a\,dx}{x^2},$$

so likewise

$$d\left(\frac{a}{t}\right) = -\frac{a\,dt}{t^2}.$$

This expression for the rate is a function of no variable except t; because, when t denotes the elapsing time, dt is not arbitrary like dx (to which any value may be assigned), but is obviously constant. In computing the numerical value of a rate, dt is taken as unity; for the date of elapsing time increases at the rate of one second per second: it is, nevertheless, necessary to retain the symbol dt in Algebraic expressions to characterize them as *differential expressions* derived from functions of t. This necessity will be more apparent as we proceed.

SPACE DESCRIBED AND VELOCITY.

66. The application of the principles of the Calculus to functions of the time is of frequent occurrence in Mechanics. Thus, if s denote the space described in the time t by a moving body under the influence of known forces, s will be a function of t, and its derivative $\frac{ds}{dt}$ will also be a function of t. Since $dt = 1$, this derivative will be equal to the rate of