

THE COMBINATION OF OBSERVATIONS

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The combination of observations by David Brunt

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DAVID BRUNT

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OF OBSERVATIONS**

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BY

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unknown quantity will be found in the first four Chapters. Many of the problems of the physicist involve one unknown only, and the first four Chapters contain all the theory that has to be considered in the discussion of such problems. The subject of Chapter VII, the adjustment of conditioned observations, has only been outlined very briefly. The fuller development of the subject, which forms the basis of the adjustment of triangulations, will be found in the works to which reference is made at the end of Chapter VII. The last four Chapters can only be regarded as mere introductions to the subjects discussed, but it was thought that their inclusion in a textbook on Least Squares would be an advantage.

I have to acknowledge my indebtedness to Mr F. J. M. Stratton, of Gonville and Caius College, Cambridge, to whose University lectures I owe most of my knowledge of the subjects discussed in this book, and upon whose notes I have drawn freely. I have also to thank Professor Eddington for many useful suggestions made while the book was in manuscript form, and for permission to extract from his lecture notes a number of interesting examples as well as some portions of the theoretical treatment.

I cannot express adequately my debt to the Cambridge University Press, for the extreme care shown in passing the book through the proof stage. Owing to my being abroad at the time, I was not able to devote as much time as was desirable to the reading of proofs, and but for the unflinching vigilance of the Press, many errors would have been allowed to pass into the text.

DAVID BRUNT.

METEOROLOGICAL SECTION, R.E.

G. H. Q.

November 23, 1916.

TABLE OF CONTENTS

CHAPTER I

ERRORS OF OBSERVATION

	PAGE
The nature of errors of observation	1
Frequency curves	6
Accuracy of the arithmetic mean	9

CHAPTER II

THE LAW OF ERROR

Hagen's proof of Gauss's error law	11
Thomson and Tait's proof	14
Generalised form of Hagen's proof	15
Form of the error curve	18
The arithmetic mean	21
Derivation of the normal law of errors from the principle of the arithmetic mean	22
Law of error of a linear function of two independent quantities	24
The median	27
Examples	28
References	28

CHAPTER III

THE CASE OF ONE UNKNOWN

Measures of precision	29
Evaluation of h and r	31
Comparison of a set of observations with theory	33
Evaluation of μ	33
Probable error and mean square error of the arithmetic mean	36
Probable error of a linear function	36
Probable error of the arithmetic mean	37
Peters' formula for r	38

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The median	27
Examples	28
References	28

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Measures of precision	29
Evaluation of h and r	31
Comparison of a set of observations with theory	33
Evaluation of μ	33
Probable error and mean square error of the arithmetic mean	36
Probable error of a linear function	36
Probable error of the arithmetic mean	37
Peters' formula for r	38

	PAGE
Examples of the adjustment of observations of one unknown	39
Probable error of any function of a number of independent quantities	48
Examples	49
Errors due to separable causes	50
Probable error when a systematic error is present	51
Correction of statistics for the effects of a known probable error of observation	53
Precision of the probable error	54
Comparison of the two formulae for r	58

CHAPTER IV

OBSERVATIONS OF DIFFERENT WEIGHT

The weighting of observations	60
Methods of weighting	63
Alternative method of evaluating the precision of the weighted mean when weighting according to probable errors	67
Examples	69
Miscellaneous Examples involving one unknown	72

CHAPTER V

THE GENERAL PROBLEM OF THE ADJUSTMENT OF INDIRECT OBSERVATIONS INVOLVING SEVERAL UNKNOWN QUANTITIES

Statement of the problem	75
Formation of the normal equations	77
Independence of the normal equations	79
Checks on the formation of the normal equations	80
Examples of the formation of normal equations	82
Solution of the normal equations	88

CHAPTER VI

EVALUATION OF THE MOST PROBABLE VALUES OF THE UNKNOWN, THEIR WEIGHTS AND PROBABLE ERRORS

Gauss's method of substitution	90
Checks in computation	91
Form of solution	94
The Doolittle method of solution	96
The method of determinants	99
Probable error of an observational equation of unit weight	100
Evaluation of probable errors of the unknowns	103

CONTENTS

ix

	PAGE
Evaluation of the weights of x, y, z , etc.	105
Examples	107
Alternative method of finding the weights of the unknowns	108
Alternative proof of the rule for finding weights of the unknowns	113
Alternative proof of the formula	
$r = 0.6745 \sqrt{\frac{[\nu v]}{n - m}}$	115
Probable error of any function of the unknowns.	116
Normal place method in the formation of observational equations	117
Testing the results of the least square solution for unusual errors, and for systematic or constant errors.	118
Miscellaneous Examples.	120

CHAPTER VII

THE ADJUSTMENT OF CONDITIONED OBSERVATIONS

Statement of the problem	124
Direct solution by substitution	125
Method of undetermined multipliers or correlates	127
The precision of the unknowns	128
References	128

CHAPTER VIII

THE REJECTION OF OBSERVATIONS 129

CHAPTER IX

ALTERNATIVES TO THE NORMAL LAW OF ERRORS

Some statistical terms	133
The methods of moments	136
Pearson's curves	138
Integration of the differential equation	141
Use of the series	
$y = A_0\phi(x) + A_3\phi'''(x) + A_4\phi^{IV}(x) + \text{etc.}$	145

CHAPTER X

CORRELATION

A simple example	148
The contingency table	150
Adjustment of the line of regression by the method of least squares	152