A TREATISE ON PLANE AND SPHERICAL TRIGONOMETRY, PP. 1-253

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A Treatise on Plane and Spherical Trigonometry, pp. 1-253 by William Chauvenet

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WILLIAM CHAUVENET

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Trieste

W. J. Hussey,
Oct. 8, 1887

A

TREATISE

ON

PLANE AND SPHERICAL

TRIGONOMETRY.

BY

WILLIAM CHAUVENET.

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PREFACE.

I HAVE in this treatise endeavored to arrange a course of trigonometrical study sufficiently extensive to enable the student to comprehend readily any applications of trigonometry he may meet with in the works of the best modern mathematicians. With this object, some topics have been introduced which are not usually found in works devoted specially to this subject.

Among those topics, the most important is the solution of the general spherical triangle, or the triangle whose sides and angles are not limited, according to the usual practice, to values less than 180°. The advantage of introducing such triangles into astronomical investigations is sufficiently shown in the applications made of them in the works of BESSEL and other German mathematicians; and especially in the *Theoria Motus Corporum Cœlestium* of GAUSS, who was the first to suggest their employment.

The subject of Finite Differences of triangles, plane and spherical, occupies a large space in Cagnoli's treatise, but has not been admitted into more recent works. It here occupies only a few pages, but no important result of

Cagnoli's Table has been omitted, while a number of the formulæ are much simpler than the corresponding ones given by him.

Although my plan embraces a much more extensive course than is contained in the text-books commonly used, I have studiously kept in view the wants of academic and collegiate classes; and have so arranged the work that a selection of subjects of immediate importance may be readily made. The more elementary portions are printed in a larger type, and are intended to form, independently of the matter in the smaller type, a connected treatise which may be studied as though it were in a separate volume.

Those who may afterwards wish to extend their knowledge will appreciate the advantage of having the higher departments of the subject treated in connection with those fundamental ones to which they are most intimately related.

W. C.

U. S. NAVAL ACADEMY,
Annapolis, Md., May 1, 1850

NOTE TO THE FOURTH EDITION.

IN this edition, besides a number of minor changes, and the correction of some typographical errors, a very important modification has been made in the solution of the equation $\tan x = p \tan y$ by series (p. 145), which was given in former editions in the usual form as stated by all writers on trigonometry. This form was discovered to lack generality, and consequently to fail in certain applications, in consequence of the omission of the arbitrary term $n\pi$ now introduced. Several subsequent investigations, depending on this, have in like manner been rectified.

W. C.

U. S. NAVAL ACADEMY, *April* 1, 1854.

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